

# Exact solutions in $\mathcal{R}^2$ SUGRA

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## Abstract

This letter is devoted to show that the bosonic sector of the  $\mathcal{R}^2$ -SUGRA in four dimensions, constructed with the F-term, admits a variety of exact and analytic solutions which include, pp- and AdS waves, asymptotically flat and AdS black holes and wormholes, as well as product spacetimes. The existence of static black holes and wormholes implies that a combination involving the Ricci scalar plus the norm of the field strength of the auxiliary two-form  $B_{\mu\nu}$ , must be a constant. We focus on this sector of the theory which has two subsectors depending on whether such a combination vanishes or not. <sup>1</sup>

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## I. INTRODUCTION

Recently, there has been renewed interest in theories of gravity with higher derivative terms, particularly, in the case of theories containing quadratic terms in the curvature (see e.g. [1]-[2] ).

It is well known that in string theory an infinite series of higher curvature correction to General Relativity arise naturally. The inclusion of some of these terms makes the gravitational theory under consideration renormalizable but the truncation of the series to an effective field theory is done at the cost of introducing a massive ghost degree of freedom. At quadratic order, this state appears due to the presence of terms which are the square of the Riemann, Ricci or Weyl tensors<sup>1</sup> [3].

The case of  $R + R^n$  gravity is particularly interesting, since after a field redefinition it can be rewritten as General Relativity with a minimally coupled scalar field with a self interaction. In the context of cosmology Starobinsky considered the simplest modification to GR, the case of  $n = 2$ , and it was shown that the effective potential for the scalar degree of freedom has a structure that is very ad hoc for the study of cosmic inflation. Indeed, it describes a slow roll transition from a de Sitter phase to a flat Minkowski phase [4], leading to a possible realization of an inflationary era in the early Universe. Even more, cosmological scenarios have been investigated embedding this model in the context of supergravity theories (see e.g. [5]).

Moreover, contrary to other higher derivative extensions,  $R + R^2$  gravity is ghost-free. The theory along with a massless graviton also propagates a massive scalar degree of freedom usually called scalaron field [6]. Even though in higher derivative modified gravity the Einstein term is usually present, recently the case of pure higher derivative theories have been investigated in various contexts [7], [8] and different features of black hole thermodynamics have been explored in [9], [10]. The specific case of  $R^2$  theory shows interesting properties.  $R^2$  gravity is ghost free and scale invariant. The scale invariance holds classically and it is expected that at quantum level the Einstein-Hilbert term is induced by broking the symmetry in a soft manner [5]. The theory is conformally equivalent to general relativity plus cosmological constant with minimally coupled scalar fields and its particle content depends

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<sup>1</sup> At the level of the field equations in  $D = 4$ , only two of these are independent since the Euler density does not contribute to the equations of motion.

on the background. For instance on a flat background the theory only propagates a scalar mode while in a curved background (de Sitter), along with the scalar state it propagates a massless graviton. The later is in agreement with the fact that for  $R \neq 0$  the theory is equivalent as we mentioned above to general relativity with cosmological constant plus minimally coupled scalar fields. For  $R = 0$  this conformal transformation becomes singular. Many of these features let the authors of [11] to revisit the supersymmetric extensions of pure  $R^2$  gravity. In its new minimal formulation, the multiplet of the  $R^2$  supergravity consists on a vierbein and its supersymmetric partner the gravitino, and the off-shell realization of supersymmetry is ensured by introducing two auxiliary fields that turn out to be a gauge field together with a two-form, both possessing the gauge symmetry.

The aim of this letter is to explore the space of solutions of the bosonic sector of this  $\mathcal{R}^2$  supergravity. We will first establish that the existence of regular static solutions imposes that a particular linear combination of the scalar curvature together with the norm of the dual field strength associated to the two-form must be a constant  $c$ . Under this restriction, the original field equations reduce to a second-order, traceless system. For a constant  $c = 0$ , the theory is no longer equivalent to a scalar tensor theory and the Einstein field equations are automatically satisfied by imposing a certain relation between the gauge field and the dual field strength. In this case, we will construct black holes and wormholes. At this particular point, pp-waves are also admissible with a null dual field. On the other hand for  $c \neq 0$ , we establish the existence of product spaces of the form  $\mathbb{R} \times H_3$  and  $dS_3 \times \mathbb{R}$  where  $H_3$  and  $dS_3$  denote respectively the three-dimensional hyperbolic and de-Sitter spaces. Interestingly enough, the field equations are also shown to support AdS-waves or Siklos spacetimes.

This paper is organized as follows: In Section II, we define the theory, providing the Lagrangian as well the field equations. In Section III, we establish that the existence of static regular solutions requires that the following combination  $R + 6H_\mu H^\mu$  must be a constant  $c$ , where  $R$  stands for the scalar curvature and  $H_\mu$  denotes the dual field strength. This latter condition notably simplifies the system for asymptotically flat or AdS black holes and wormholes, reducing the field equations to a traceless, second order system. Section IV is devoted to present some interesting solutions with non-vanishing  $c$  while in Section V we construct static black holes and wormholes that correspond to solutions with  $c = 0$ . In the last Section, we provide some further comments.

## II. THE THEORY

In this section, we present the bosonic sector of the  $F$ -terms of the  $\mathcal{R}^2$  supergravity (see [11]). The bosonic action is described by the metric and the two auxiliary gauge fields, and is given by<sup>2</sup>

$$I[g_{\mu\nu}, A_\mu, B_{\mu\nu}] = \int \sqrt{-g} d^4x \left( \frac{1}{8g^2} (R + 6H_\mu H^\mu)^2 - \frac{1}{4g^2} F_{\mu\nu} (A_\rho^-) F^{\mu\nu} (A_\rho^-) \right), \quad (1)$$

where

$$A_\mu^- = A_\mu - 3H_\mu, \quad (2)$$

and

$$H_\mu = -\frac{1}{3!} \epsilon_{\mu\nu\rho\sigma} H^{\nu\rho\sigma}. \quad (3)$$

Here, the three-form  $H_{\mu\nu\lambda}$  is defined as the field strength of the auxiliary two form  $B_{\mu\nu}$ , i.e.,  $H_{\nu\rho\sigma} := \partial_\mu B_{\nu\rho} + \text{cyclic perm.}$  The local existence of  $B_{\mu\nu}$  is ensured provided the condition  $\nabla_\mu H^\mu = 0$  holds.

Note that the action is invariant under the rigid scale transformation

$$g_{\mu\nu} \rightarrow \omega g_{\mu\nu}, \quad A_\rho \rightarrow A_\rho, \quad H_\mu \rightarrow H_\mu, \quad (4)$$

where  $\omega$  is an arbitrary non-vanishing constant.

The corresponding field equations coming from the variations of the action (1) with respect to  $g_{\mu\nu}$ ,  $B_{\mu\nu}$  and  $A_\mu$ , reduce to the following set of equations

$$\left[ g_{\mu\nu} \square - \nabla_\mu \nabla_\nu + R_{\mu\nu} - \frac{1}{4} g_{\mu\nu} R + 6 \left( H_\mu H_\nu - \frac{1}{4} g_{\mu\nu} H_\alpha H^\alpha \right) \right] (R + 6H_\alpha H^\alpha) = 2T_{\mu\nu}^M (A^-), \quad (5)$$

$$\nabla^\mu [(R + 6H_\alpha H^\alpha) H_{\mu\beta\gamma}] = 0, \quad (6)$$

$$\nabla_\mu F^{\mu\nu} (A^-) = 0, \quad (7)$$

with  $T_{\mu\nu}^M$  being the Maxwell energy-momentum tensor defined as

$$T_{\mu\nu}^M (A^-) := F_{\mu\alpha} (A^-) F_\nu{}^\alpha (A^-) - \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} (A^-) F^{\alpha\beta} (A^-). \quad (8)$$

In the next section we will establish that the existence of regular, static solutions requires the following combination  $R + 6H_\alpha H^\alpha$  to be a constant.

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<sup>2</sup> As shown in [11], generically, this theory is conformally equivalent to a second order theory, and therefore free of Ostrogradski instabilities.

### III. ON STATIC AND REGULAR SOLUTIONS

Let us consider the following static ansatz for the metric

$$ds^2 = -\lambda^2(x) dt^2 + h_{ij}(x) dx^i dx^j , \quad (9)$$

where  $h_{ij}(x)$  is a spacelike metric and  $\lambda(x)$  depends only on the spacelike coordinates  $x^i$ . This metric may cover the domain of outer communications of a static black hole or even a global metric on a wormhole geometry. In the former case, the function  $\lambda(x)$  vanishes at the horizon while for the wormhole solution the function is non-vanishing everywhere. Considering the trace of equation (5) and multiplying the result by  $\lambda(x)(R + 6H_\mu H^\mu)$  one obtains

$$D_i (\lambda(x)(R + 6H_\mu H^\mu) D^i (R + 6H_\mu H^\mu)) - \lambda(x) D_i (R + 6H_\mu H^\mu) D^i (R + 6H_\mu H^\mu) = 0 , \quad (10)$$

where  $D_i$  is the covariant derivative intrinsically defined by  $h_{ij}(x)$ . Integrating this result on a  $t$ -constant (spacelike) hypersurface and disregarding boundary terms one obtains

$$R + 6H_\mu H^\mu = c , \quad (11)$$

where  $c$  is a constant. To be able to drop the boundary terms, in the black hole geometry we have to require a certain asymptotic behavior at infinity, while to do so in the case of the wormhole geometry we have to require such asymptotic behavior to be valid at both asymptotic regions.

Note that by performing a rigid scale transformation, according to the symmetry (4), we can normalize the constant  $c$  to  $\pm 1$  or 0.

The constraint (11) considerably simplifies the original field equations (5) and it is clear that the cases  $c = 0$  and  $c \neq 0$  must be treated separately. Indeed, for a non-vanishing  $c$ , the equations (5-7) yields a reduced system defined by the following second order equations

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R + 6 \left( H_\mu H_\nu - \frac{1}{4}g_{\mu\nu}H_\alpha H^\alpha \right) = \frac{2}{c}T_{\mu\nu}^M(A^-) , \quad (12)$$

$$R + 6H_\mu H^\mu = c , \quad (13)$$

$$\nabla_\mu H^{\mu\beta\gamma} = 0 , \nabla_\mu F^{\mu\nu}(A^-) = 0 , \quad (14)$$

with the further requirement that  $\nabla_\mu H^\mu = 0$ .

Even though this reduced system is of second order, it is non-trivial to find general solutions for spacetimes with a given symmetry. Nevertheless we have been able to construct some scattered solutions that we report and analyze in the next sections. On the other hand, if the constant  $c$  vanishes, besides the gravitational constraint  $R + 6H_\mu H^\mu = 0$ , one has that the theory reduces to Maxwell theory for  $A_\mu^-$  with the further constraint of vanishing energy-momentum tensor, i.e.

$$\nabla_\mu F^{\mu\nu} (A^-) = 0, \quad T_{\mu\nu}^M (A^-) = 0. \quad (15)$$

For the static ansatz (9), this implies that  $A_\mu$  and  $3H_\mu$  are equal, up to a total derivative.

We separate the solutions we have found in two families depending on whether  $c$  in (11) vanishes or not.

#### IV. *AdS* WAVES AND PRODUCT SPACES WITH NON-VANISHING $c$

We now turn our analysis in looking for solutions of the reduced field equations (12)-(13)-(14) for which the constant  $c \neq 0$ . Since *AdS*-waves as well as products of constant curvature spacetimes have constant scalar invariants, it is natural to explore the possibility for such spacetimes to be solutions of our reduced field equations. In addition, we assume that the  $U(1)$  gauge field  $A_\mu$  and the dual field  $H_\mu$  are proportional

$$A_\mu = 3H_\mu, \quad (16)$$

condition which automatically ensures that the Maxwell equations are satisfied, and also allows to decouple the Maxwell part from the Einstein equations. Indeed, in this case the latter reduce

$$R_{\mu\nu} - \frac{1}{4}g_{\mu\nu}R + 6\left(H_\mu H_\nu - \frac{1}{4}g_{\mu\nu}H_\alpha H^\alpha\right) = 0, \quad (17)$$

and where the divergence-free condition  $\nabla_\mu H^\mu = 0$  must also hold.

- The product spaces  $\mathbb{R} \times H_3$ :

Let us consider the product spaces  $\mathbb{R} \times H_3$  where  $H_3$  denotes the three-dimensional hyperbolic space. In this case, a solution is given by

$$ds^2 = -dt^2 + \frac{l^2}{y^2} (dx^2 + dy^2 + dz^2), \quad H = \pm \frac{\sqrt{3}}{3l} dt. \quad (18)$$

This corresponds to a solution of (17) with  $c = -8/l^2$ , and since the equations are local, one could as well consider quotients  $H_3/\Gamma$ , where  $\Gamma$  is a freely acting, discrete subgroup

of  $SO(3,1)$ . This identification might have some effect on the possible global existence of Killing spinors.

- The product spaces  $dS_3 \times \mathbb{R}$ :

As before, we show that the product of the three-dimensional de Sitter space with  $\mathbb{R}$  provide a simple solution of the reduced equations with non-vanishing  $c$ . Indeed, in this case, the solution reads

$$ds^2 = - \left(1 - \frac{r^2}{l^2}\right) dt^2 + \frac{dr^2}{1 - \frac{r^2}{l^2}} + r^2 d\phi^2 + dz^2, \quad H = \pm \frac{\sqrt{3}}{3l} dz, \quad (19)$$

and corresponds to a solution with a positive constant  $c = 8/l^2$ . Note that this solution can be Wick rotated yielding a new Euclidean solution which reduces locally to  $S^3 \times \mathbb{R}$ . The fields  $H$  and  $A$  remain invariant under the Wick rotation.

- AdS-waves:

As mentioned before, the AdS waves present the advantage that all their scalar invariants are constants making these configurations possible interesting candidates for the sector of the theory we are exploring. Note that such configurations have been shown to be solutions of Einstein gravity with a nonminimal scalar field [12] as well as for quadratic corrections of the Einstein gravity [13, 14]. The line element of the AdS waves can be parameterized as follows

$$ds^2 = \frac{l^2}{y^2} (-F(u, y, x) du^2 + 2du dv + dy^2 + dx^2), \quad (20)$$

which clearly emphasize that these spacetimes can be obtained from AdS through a Kerr-Schild transformation. This allows to interpret the AdS waves as exact gravitational waves propagating on the AdS space, and consequently their matter source must behave as a pure radiation field. This last condition may be satisfied requiring the dual field  $H_\mu$  to be along the retarded time  $u$  as

$$H_\mu dx^\mu = f(u, y, x) du, \quad (21)$$

where  $f$  is a function depending on all the coordinates except the null one  $v$ . Note that the divergenceless condition of  $H_\mu$  is automatically satisfied in this case. The field equations (17) with an Ansatz of the form (20-21) will be fulfilled provided that the following equation

$$\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} - \frac{2}{y} \frac{\partial F}{\partial y} + 12f^2 = 0, \quad (22)$$

holds. In this case  $R = -12/l^2$  and since  $H_\mu$  is null the constant  $c = -12/l^2$  is non-vanishing. This means that for any null field of the form (21), one is able in principle to find

AdS wave configurations provided that the structural function  $F$  satisfies the equation (22). For example, in the case with  $f$  being a constant, this equation may be easily integrated yielding

$$F = a(u) (x^2 + y^2) + b(u) x + c(u) y^3 + 6f^2 y^2 + d(u) , \quad (23)$$

where  $a$ ,  $b$ ,  $c$  and  $d$  are arbitrary functions of the retarded time  $u$ .

Just to conclude this section, we would like to mention that pp waves are also solutions of the equations (22) with a null dual field, but in this case they correspond to a solution with a vanishing constant  $c$ .

The solutions we have just mentioned have non-vanishing  $c$ , and we have presented cases with  $c$  being positive and negative. In the next section we present solutions with vanishing  $c$ .

## V. SOLUTIONS WITH VANISHING $c$ . BLACK HOLES, WORMHOLES AND BEYOND

We now look for solutions for which the integration constant  $c = 0$ , and we first consider the following spherically symmetric Ansatz

$$ds^2 = -f(r) dt^2 + \frac{dr^2}{f(r)} + (r^2 + a^2) d\Omega_2^2 , \quad (24)$$

where  $d\Omega_2$  stands for the line element of the two-sphere. If  $f(r)$  is non-vanishing, this metric describes a wormhole geometry where the surface  $r = 0$  represents a traversable wormhole throat. Geodesics with vanishing angular momentum are pulled towards the minimum of the function  $f(r)$ . On the other hand, if  $f(r)$  vanishes the metric (24) may represent a black hole geometry with an event horizon located at  $r = r_+$  such that  $f(r_+) = 0$ .

The whole system of equations (5-7) are solved by

$$A_\mu = 3H_\mu, \quad H = \frac{\chi(\theta)}{r^2 + a^2} dt + \frac{\chi(\theta)}{f(r)(r^2 + a^2)} dr + \frac{\sqrt{2}\sqrt{r^2 + a^2}}{l} \sin(\theta) d\phi , \quad (25)$$

provided

$$\frac{d^2 f(r)}{dr^2} + \frac{4r}{r^2 + a^2} \frac{df(r)}{dr} + \frac{2(r^2 + 2a^2)}{(r^2 + a^2)^2} f(r) = \frac{2(6r^2 + 6a^2 + l^2)}{l^2(r^2 + a^2)} . \quad (26)$$

Here  $H_\mu H^\mu = \frac{2}{l^2}$  and  $H_\mu$  is divergenceless, ensuring the local existence of the auxiliary two-form  $B_{\mu\nu}$ . The function  $\chi(\theta)$  is arbitrary, and in this case the constant  $c$  in equation (11) vanishes.



The asymptotic expansion of (26) at  $r \rightarrow \pm\infty$ , reveals that the leading term in  $f(r)$  behaves as  $f(r) \rightarrow \frac{r^2}{l^2}$ , and therefore the solutions of (26) will provide us with asymptotically locally AdS spacetimes. The general solution of (26) can be written in terms of hypergeometric functions  ${}_2F_1$ , and is given by

$$f(r) = \frac{e^{-i\sqrt{3}\tan^{-1}(\frac{r}{a})}}{6a(a^2 + r^2)^{3/2}} \left( -(a^2 + r^2) \left( 6ae^{i\sqrt{3}\tan^{-1}(\frac{r}{a})} \sqrt{a^2 + r^2} \left( 4a^2 + r^2 + 1 + c_1 \frac{e^{i\sqrt{3}\tan^{-1}(\frac{r}{a})}}{\sqrt{a^2 + r^2}} \right) + \sqrt{3}ic_2 \right) \right. \\ \left. + 2a^2(1 + 3a^2)e^{i\sqrt{3}\tan^{-1}(\frac{r}{a})}(a + ir)\sqrt{a^2 + r^2} \left( (3 + \sqrt{3}) {}_2F_1 \left[ 1, \frac{1}{2}(1 - \sqrt{3}), \frac{1}{2}(3 - \sqrt{3}), 1 - \frac{2a}{a - ir} \right] \right. \right. \\ \left. \left. - (-3 + \sqrt{3}) {}_2F_1 \left[ 1, \frac{1}{2}(1 - \sqrt{3}), \frac{1}{2}(3 - \sqrt{3}), 1 - \frac{2a}{a + ir} \right] \right) \right) \right),$$

where  $c_1$  and  $c_2$  are two integration constants. Since the equation (26) is a linear, non-homogeneous, ordinary differential equation, one can take the real part of this solution to obtain a function that defines a metric. Depending on the values of the integration constants the solution may describe black hole or wormhole configurations as shown in Figure 1. These solutions extend the solutions recently found in [15] to the asymptotically AdS case.

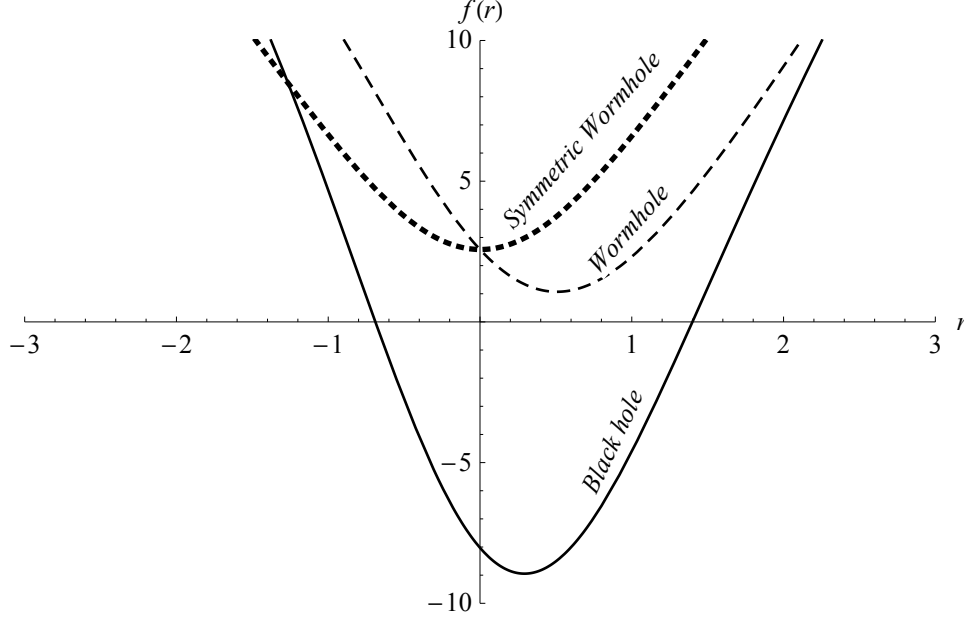


FIG. 1: Real part of  $f(r)$  vs  $r$  for different values of the integration constants.

The black line in Figure 1, describes a regular black hole, which is asymptotically AdS. The first zero of the function  $f(r)$  (from right to left) defines the event horizon, while the second one defines a Cauchy horizon. The dashed line in Figure 1, describes a wormhole which is asymptotically AdS at both asymptotic regions. The lapse function has a minimum

at a certain point which in general differs from the location of the throat (located at  $r = 0$ ). As observed for example in [16] and [15], this has an interesting effect on the geodesics with angular momentum since gravity pulls geodesics with vanishing angular momentum towards the minimum of the function  $f(r)$  while the centrifugal contribution reverses its direction at the throat. Therefore, there is a region between the throat and the minimum of the function  $f(r)$  on which both contributions on geodesics with angular momentum point in the same direction and therefore the geodesic flow obligates a particle to go out from such region.

As a second example of solutions with  $c = 0$ , one can consider as well the most general Petrov type D spacetime in four dimensions which is the Plebanski-Demianski spacetime, whose metric is given by

$$ds^2 = \frac{1}{(1-pq)^2} \left[ (p^2 + q^2) \left( \frac{dp^2}{X(p)} + \frac{dq^2}{Y(q)} \right) + \frac{X(p)}{p^2 + q^2} (d\tau + q^2 d\sigma)^2 - \frac{Y(q)}{p^2 + q^2} (d\tau - p^2 d\sigma)^2 \right]. \quad (27)$$

In this case, one can show that the system (5-7) is fulfilled for this metric with

$$H = \frac{(pq - 1)}{l} \sqrt{\frac{2X(p)}{p^2 + q^2}} dt + \frac{q^2}{l} \sqrt{\frac{2X(p)}{(p^2 + q^2)(1 - pq)^2}} d\sigma, \quad (28)$$

where  $X(p)$  is positive definite. Here  $H_\mu H^\mu = 2/l^2$  and the metric functions  $X$  and  $Y$  are given by

$$X(p) = x_0 + x_1 p + x_2 p^2 + x_3 p^3 + x_4 p^4, \quad (29)$$

$$Y(q) = - \left( x_4 - \frac{1}{l^2} \right) - x_3 q - x_2 q^2 - x_1 q^3 - \left( x_0 - \frac{1}{l^2} \right) q^4. \quad (30)$$

## VI. FURTHER COMMENTS

Here, we have considered the bosonic sector of the  $\mathcal{R}^2$ -SUGRA in four dimensions for which we have obtained a variety of exact and analytical solutions. We have first shown that requiring the existence of regular static solutions, the space of configurations is such that the following combination  $R + 6H_\alpha H^\alpha$  must be a constant  $c$ .

Solutions with non-vanishing  $c$  are provided by AdS waves or product spaces of the form  $\mathbb{R} \times H_3$  and  $dS_3 \times \mathbb{R}$ . At this point, one may note a certain analogy with the standard eleven-dimensional supergravity whose purely bosonic field equations, given by

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = \frac{1}{12} \left( F_{\mu\nu}^2 - \frac{1}{8} g_{\mu\nu} F^2 \right),$$

are quite similar to our reduced field equations (17). Here  $F$  is the field strength associated to the three-form with  $F_{\mu\nu}^2 = F_{\mu\alpha\beta\sigma}F_{\nu}^{\alpha\beta\sigma}$  and  $F^2 = F_{\mu\alpha\beta\sigma}F^{\mu\alpha\beta\sigma}$ . This analogy with the standard eleven-dimensional supergravity can also be extended to the spectrum of solutions where our product space solutions are similar to the Freund-Rubin solutions  $AdS_4 \times S_7$  and  $AdS_7 \times S_4$  of eleven-dimensional supergravity. A natural task to explore would be to determine the amount of supersymmetry preserved by these backgrounds by solving the Killing spinorial equation, which is explicitly given in [17]. We leave this task for future work.

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